PRODUCTION RECONCILIATION FOR MINERAL RESOURCE MODELLING IN A PORPHYRY COPPER GOLD DEPOSIT
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The Great man

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Production reconciliation of a multivariate uniform conditioning technique for mineral resource modelling in a porphyry copper gold deposit
Outline

• Introduction.

• Methodology
  – Recap of Uniform Conditioning
  – Localized Multivariate Uniform Conditioning

• Case study.

• Production reconciliation case study, based on a porphyry copper gold deposit in Peru.

• Conclusions.
Introduction

- The extension of Uniform Conditioning (UC) techniques to the multivariate case is available by using the Discrete Gaussian Model (DGM).
- It is based on the use of correlations between different variables and one “main” variable used for selecting the selective mining unit’s (smu’s).
- The grade tonnage estimated by UC within panels can then be assigned to individual smu’s by generalizing the Localized Uniform Conditioning method to the multivariate case.
- The objective of the paper is to provide a reconciliation of the long-term MUC/LMUC mineral resources model, which is invariably based on drilling data on a relatively large grid, to the corresponding production blast hole grade control model, as well as with the final plant production.
UC Methodology

- We estimate for a selection block v:
  - Ore: \( T(z) = 1_{Z_1(v) \geq z_1} \)
  - Main Metal: \( Q_1(z_1) = Z_1(v)1_{Z_1(v) \geq z_1} \)
    (to be multiplied by block tonnage = volume x density)
- For a second element we want also:
  - Secondary Metal: \( Q_2(z_1) = Z_2(v)1_{Z_1(v) \geq z_1} \)
In the univariate case, the change of support is based on the three assumptions (DGM):

- $\mathbb{E}[Z(v)] = \mathbb{E}[Z(x)] = m$

- Krige’s relationship: $D^2(v|\mathcal{D}) = D^2(0|\mathcal{D}) - \gamma(v,v)$

- Cartier relationship: $\mathbb{E}[Z(x)|Z(v)] = Z(v)$
DGM for change of support

• The block distribution is modelled by the block anamorphosis
  \[ Z(v) = \Phi_r[Y_v] \]

• The point and block anamorphosis are related through the integral relation:
  \[ \Phi_r(y) = \int \Phi(ry + \sqrt{1-r^2u}) g(u) \, du \]

• The change of support coefficient \( r \) is calculated by means of the Krige’s relationship:
  \[ \text{Var}[Z(v)] = \text{Var}[Z(x)] - \gamma(v, v) \]
Multivariate UC

Additional assumptions are:

- $Z_1(v)$ conditional to $Z_1(V)^*$ is independent of the other element grades of the panel. The UC estimates for the main variable are the same as in the univariate case.

- $Z_1(v)$ and $Z_i(v)$ conditional to $(Z_1(V)^*, Z_i(V)^*)$ are independent of the other element grades of the panels. The multivariate case reduces to a multi-bivariate case.
• Distribution of \( Z_i(v) \) for a generic block \( v \) in panel \( V \) is conditioned by \( Z_i(V)^* \).

• We want at zero cutoff:
  \[
  E[Z_i(v) \mid Z_i(V)^*] = Z_i(V)^*
  \]
  so \( Z_i(V)^* \) is implicitly assumed to be conditionally unbiased:
  \[
  E[Z_i(V) \mid Z_i(V)^*] = Z_i(V)^*.
  \]

• UC estimates:

  \[
  \text{Ore} \quad \left[ T_V(z) \right]^* = E[1_{Z_1(v) \geq z} \mid Z_1(V)^*]
  \]

  \[
  \text{Metal} \quad \left[ Q_{2V}(z) \right]^* = E[Z_2(v)1_{Z_1(v) \geq z} \mid Z_2(V)^*]
  \]
Localized Uniform Conditioning

• UC consists of estimating the grade distribution on smu support within a panel, conditioned to the estimated panel grade, usually based on Ordinary Kriging (OK).
• In this case study, Simple Co-Kriging (SK) with local mean has been used to condition the panel grades, due to the inefficiency of the OK Co-kriging panel estimates, typical of new mining projects, which are invariably based on drilling data on a relatively large grid.
• Localized post-processing of Multivariate Uniform Conditioning, aimed at localizing the recoverable grade tonnage estimates for mine planning.
• The localization aspect consists of assigning to each block, or smu, an unsmoothed recoverable grade estimate as proposed by Abzalov(2006).
Extension to multivariate case:

The metal quantity of the secondary variables are obtained from multivariate UC, i.e. the tonnages and the related cut-offs are dependent only upon the main variable. Thus, the mean grades of secondary variables can be interpolated within the same intervals as those of the main variable.
Case Study: Geology and database

- The mineralization is found in intrusive rocks within a sedimentary host environment.
- Oxidation, weathering, leaching and subsequent secondary enrichment has led to the formation of four mineral domains with different metallurgical characteristics.
- Sulphide mineralization occurs in three main domains; the mixed domain, the supergene domain and the hypogene domain.
- The production reconciliations presented in this study, cover mainly the supergene and hypogene domains, which have significant economic importance on the mine.
- The variables studied were total gold (AUTOT), total copper (CUTOT) and Net Smelter Return (NSR).
Geology Modelling (Porphyry Cu & Au Ore body)

A view of the deposit showing geological domains – Cerro Corona, South America
Case Study: Geology and database

- The resource drilling data grid spacing were on average up to 50m x 100m.

- These were composited on a 2m basis, and were used to derive the LMUC estimates.

- The initial MUC’s were based on simple co-kriging of 40m x 40m x 10m panels, assuming 10m x 10m x 10m smu’s.
Case Study: porphyry copper gold deposit in Peru.

Drill-hole layout of the Annulus domain
Case Study: porphyry copper gold deposit in Peru

- Two economic elements are considered: total gold and total copper (AUTOT, CUTOT).
- Using economic parameters, both elements are combined into the Net Smelter Return (NSR).

<table>
<thead>
<tr>
<th></th>
<th>CUTOT</th>
<th>AUTOT</th>
<th>NSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUTOT</td>
<td></td>
<td>0.69</td>
<td>0.88</td>
</tr>
<tr>
<td>AUTOT</td>
<td>0.69</td>
<td></td>
<td>0.95</td>
</tr>
<tr>
<td>NSR</td>
<td>0.88</td>
<td>0.95</td>
<td></td>
</tr>
</tbody>
</table>

Matrix of coefficients of correlation between 3 variables on 2m composites.
In addition to the Resource drilling data, a comprehensive 6m x 5m blast hole data grid was available from mining. The blast hole data were not used for the MUC/LMUC Resource.

These were used as the follow-up “actual” block values for judging the comparative efficiency of the MUC/LMUC estimates.

Reconciliation with Plant production was also conducted.

Reconciliations were computed on monthly, quarterly and on an annual basis.
The efficiency of the reconciliations is measured on the basis of the spreads of percentage errors, defined as follows:

\[ \text{Percentage Error} = (\text{Actual}/\text{Estimate} - 1) \times 100\% \]

- Actual represents Plant production, or in-situ grade control block estimates, based on 6m x 5m blast hole data.
- Estimate is the corresponding LMUC Resource estimates before production.
Support correction is achieved with sill normalization

<table>
<thead>
<tr>
<th></th>
<th>NSR</th>
<th>CUTOT</th>
<th>AUTOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Punctual Variance (Anamorphosis)</td>
<td>276.117</td>
<td>0.08</td>
<td>0.528</td>
</tr>
<tr>
<td>Variogram Sill</td>
<td>270.45</td>
<td>0.076</td>
<td>0.536</td>
</tr>
<tr>
<td>Gamma (v,v)</td>
<td>128.191</td>
<td>0.045</td>
<td>0.212</td>
</tr>
<tr>
<td>Real Block Variance</td>
<td>147.926</td>
<td>0.035</td>
<td>0.316</td>
</tr>
<tr>
<td>Real Block Support Correction (r)</td>
<td>0.7754</td>
<td>0.69</td>
<td>0.8285</td>
</tr>
<tr>
<td>Kriged Block Support Correction (s)</td>
<td>0.7754</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Kriged-Real Block Support Correction</td>
<td>1</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Main-Secondary Block Support Correction</td>
<td>---</td>
<td>0.8733</td>
<td>0.9804</td>
</tr>
</tbody>
</table>
Case Study: porphyry copper gold deposit in Peru

- In providing the co-kriging panel conditioning estimates required for the MUC/LMUC, significant conditional biases were observed with Ordinary co-kriging (OK), as demonstrated by the large negative kriging efficiencies (KE) and poor slopes of regression associated with a substantial number of the OK based estimates.
- The inherent conditional biases as observed for the OK estimates are as a result of the limited available Resource data.
- As a result, Simple co-kriging with local means was used for the panel conditioning.
Case Study: porphyry copper gold deposit in Peru

- The LMUC approach provides smu grades with a variability closer to the actual variability.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated LMUC Dispersion Variance</th>
<th>&quot;Actual&quot; Dispersion Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>Copper</td>
<td>0.08</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table II: SMU Dispersion variance of “Actual” versus LMUC estimates

Panel estimates

Smu’s kriged estimates

Smu’s LMUC estimates
Monthly Production Reconciliation

- Provides the production reconciliation of the monthly LMUC Resource estimates with the corresponding Plant results.
- The reconciliation results are provided on the basis of the spreads of the percentage errors.
- The lower and upper 10% confidence intervals have been read directly off the histogram of the percentage of errors, as observed over the production period.
- The analyses of the spreads of the monthly percentage errors show upper and lower 10% confidence limits of -12%/+10%, -6% /+14% and -8%/+8% respectively for tonnes, gold and copper grades respectively.

<table>
<thead>
<tr>
<th>Tonnes Limits</th>
<th>Grade Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tonnes</td>
<td>Gold</td>
</tr>
<tr>
<td>Lower 10%</td>
<td>Upper 10%</td>
</tr>
<tr>
<td>-12%</td>
<td>10%</td>
</tr>
</tbody>
</table>
Distributions of Percentage Errors

Tonnes, Au and Cu grades for monthly reconciliation
(Resource model vs Plant)
Reconciliation with Production for different Periods

- Distribution of percentage errors between Resource model and Plant production over various production periods.
- The results further show percentage errors of +6%/+2%/-7% on a quarterly (i.e. 3 monthly) basis for tonnes, gold and copper grades respectively.
- Over an annual production period, the observed percentage errors were –1%/+3%/-1%, demonstrating the narrowing of the observed percentage errors over the annual period.

<table>
<thead>
<tr>
<th>Period</th>
<th>Tonnes</th>
<th>Grade</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gold</td>
<td>Copper</td>
</tr>
<tr>
<td>Quarterly</td>
<td>6%</td>
<td>2%</td>
<td>-7%</td>
</tr>
<tr>
<td>6-monthly</td>
<td>7%</td>
<td>5%</td>
<td>-2%</td>
</tr>
<tr>
<td>Annually</td>
<td>-1%</td>
<td>3%</td>
<td>-1%</td>
</tr>
</tbody>
</table>
Reconciliation : MUC/LMUC vs Grade Control

- Reconciliation between Resource models and the Grade control model
- The table below shows that the Resource models compare well with the Grade control model (also when compared to the internationally accepted 15% errors: e.g. for annual production period – see Stoker, 2011).

<table>
<thead>
<tr>
<th>Period</th>
<th>Tonnes</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Gold</td>
</tr>
<tr>
<td>2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 months</td>
<td>-0.6</td>
<td>9.6</td>
</tr>
<tr>
<td>6 months</td>
<td>-0.6</td>
<td>6.5</td>
</tr>
<tr>
<td>Annual</td>
<td>-0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 months</td>
<td>-0.1</td>
<td>2.5</td>
</tr>
<tr>
<td>6 months</td>
<td>-0.4</td>
<td>6.5</td>
</tr>
</tbody>
</table>
• However, the individual LMUC selective mining block estimates, based on Simple co-kriging conditioning (SK), show some conditional biases as reflected by the slope of regressions of 0.7/0.52 for Au and Cu respectively.

• The conditional biases are as a result of the limited available resource data used for the LMUC resource estimates.

• Additional significant conditional biases (i.e. significantly higher than that of SK co-kriging above) were observed with Ordinary co-kriging (OK) conditioning.
Regression of LMUC vs Grade Control
Conclusions

• Gaussian models (in this case MUC) used for calculating recoverable resources provide consistent results in modelling the change of support and the information effect in the multivariate case.

• The production reconciliation results show the overall advantage gained by using MUC/LMUC estimates based on SK co-kriging as demonstrated by the narrow spreads of percentage errors.

• The central 80 per cent confidence limits of the monthly production errors were -12%/+10%, -6%/+14% and -8%/+8% respectively for tonnes, gold and copper grades respectively.

• The narrowing of the observed confidence limits are also observed as shown by the reduced observed average percentage errors of –1%/+3% for the plant production reconciliations on a macro or long term production basis.

• The study further showed that on a local production scale (and especially for short to medium term planning), regression effects and conditional biases were still evident with the assigned LMUC individual SMU estimates.

• Significant conditional biases were particularly evident with the Ordinary co-kriging estimates which were mainly due to the limited data that were available for the LMUC Resource estimates.

• In this regard, the Simple co-kriging estimates based on local means, showed more efficient panel conditioning estimates for the purpose of the MUC/LMUC resource assessment and the reconciliations.
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